WHAT IS BUCKLING?

by

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INTRODUCTION

The frequent use of the term "buckling" baffles some of our friends in the engineering, metallurgical, and chemical branches of our technology, just as we reactor physicists are often perplexed by metallographic slides or by process stream terminology.

The barrier against the understanding of buckling is partly semantic and partly mathematical. The first difficulty arises from our careless use of the same word for two fundamentally different concepts: geometric buckling and material buckling. The second arises from the fact that a concise and elegant treatment of the subject involves the use of differential equations. At the risk of being somewhat long-winded, I am taking an "operational" approach to make the two kinds of buckling plausible without higher mathematics.

The reader is presumed to know that the neutrons we are concerned with are set free as a result of nuclear fission inside a reactor, but perish after an erratic journey by being caught in a nucleus, fissionable or not, inside or outside the reactor.

1. The Multiplication Constant

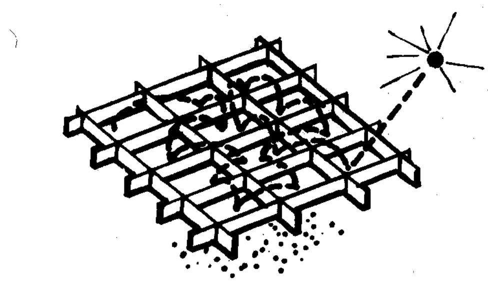
When a neutron chain reaction continues at a constant rate it is because enough of the neutrons born in a given number of unrelated fissions* survive competing hazards to give rise to the same number of new fissions. This survival is threatened by two kinds of accidents that may terminate the useful life of a neutron prematurely. One is leakage from the reactor, the other is non-productive absorption within the reactor.

The ratio of fissions in two successive related generations is called k, the multiplication constant. If k is less than one, the offspring are less numerous than the progenitors and the fission rate declines with time. If k is greater than one, fissions become more frequent in time and the reactor power increases. Whether a given type of lattice proposed by an engineer will support a chain reaction within an enclosure of his choice (i.e., whether or not k can be ≥ 1) is a crucial question that must be answered by the physicist before the engineer gets involved in detailed design.

2. Leakage from the Reactor; Geometric Buckling

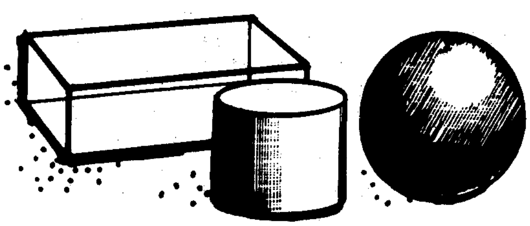
The answer depends, in part, on the amount of leakage of neutrons from the reactor. If the lattice were infinite, there would be no leakage. Thus, it is convenient to write k as a product of two terms, the multiplication constant of an infinite lattice of the proposed composition, k_∞, and the fraction of the neutron offspring that *does not* leak from the finite lattice. The latter term depends on properties of the lattice and also on the size and shape of the reactor. These dependencies can, in fact, again be split into two separate factors, one of which relates to the lattice only and the other to the geometric shape only. The first has to do with the average distance traveled by a neutron in the lattice from its place of birth to its place of death. Only neutrons that are born near the surface can escape from the reactor. Conversely, neutrons that are born at a distance inside the surface of the reactor that is substantially greater than the average traveling, or "migration" distance within the lattice will not get to the surface and, hence, will not leak out. Actually, the first factor in the leakage term turns out to be the average square of the distance traveled by the neutrons during their life within the lattice, and is called the migration area, M^2. The greater M^2, the greater is the depth from which neutrons can leak and therefore the greater is the total leakage from the reactor.

*For example, in all fissions occurring within a given time interval that is short compared to the lifetime of a neutron in the reactor.



The second factor entering into the non-leakage term is related to the size and shape of the reactor, only. It is possible to combine all pertinent information on size and shape in a single expression. This turns out to vary inversely as the second power of the characteristic dimensions of the reactor. It is called the geometric buckling, B_g^2. Formulae for the geometric buckling of the simplest shapes are as follows:

Shape	B_g^2	Definition of Symbols
Parallelepiped	$\pi^2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$	a, b, c = edges
Cylinder	$\pi^2/h^2 + 2.405^2/r^2$	h = height; r = radius
Sphere	π^2/R^2	R = radius



In principle, a B_g^2 can be determined for any size or shape. It is a purely geometric procedure. The geometric buckling may be expressed in units of cm^{-2}. In practice, this unit is too large, and smaller units are used. Some people use m^{-2}($= 10^{-4}\,cm^{-2}$), others use $10^{-6}\,cm^{-2}$, the *microbuck*.

It can be shown that the product of the two factors, M^2 and B_g^2, represents the number of neutrons that leak from the reactor for each neutron that dies within the reactor. Hence, the fraction of all lost neutrons that are lost through leakage is

$$\frac{M^2 B_g^2}{1 + M^2 B_g^2}$$

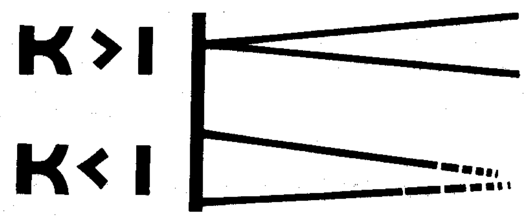
and the fraction of neutrons that *do not* leak is

$$\frac{1}{1 + M^2 B_g^2} .$$

The multiplication constant can now be written in the form

$$k = \frac{k_\infty}{1 + M^2 B_g^2} . \qquad (1)$$

Once the lattice and reactor size and shape are chosen, the crucial question, whether or not k can be ≥ 1, is thus reduced to the determination of M^2 and k_∞, both of which are properties of the (infinite) lattice. M^2 can be calculated or measured, more or less directly. However, the evaluation of k_∞ involves a number of separate steps. These steps will be discussed in the following section. After reading that section, the reader may wish that the determination of M^2 and of all the quantities entering k_∞ be simplified and replaced by a single concept and measurement. This wish will be fulfilled when we get to the section on *Material Buckling*.



3. Survival Inside the Reactor

Not all of the neutrons that escape leakage from the reactor contribute to the chain reaction by causing fission. Many are captured in the non-fissionable nuclei that make up the structural material, the moderator and other substances present. Even when a neutron is captured in a fissionable nucleus, there is a fair chance that this nucleus will not undergo fission. In that case, a neutron is withdrawn from the chain reaction by the fuel itself.

The average number of neutrons created in a fission event is designated by ν, (= 2.43 for U^{235}). To arrive at a value of k_∞ for a proposed lattice, this number ν must be multiplied by the probability that a neutron will escape capture by a non-fissionable nucleus and by the probability that a fissionable nucleus, having captured the neutron, will undergo fission. The latter probability is usually written as

$$\frac{1}{1 + \alpha}$$

where α is the probability ratio, capture to fission, in the fissionable nucleus.

Thus, $k_\infty = \nu x$ (capture escape probability) $\times \frac{1}{1+\alpha}$.
Since both ν and α are nuclear properties of the fuel and not directly properties of the lattice, they are often represented by a common symbol, *the neutron reproduction factor*:

$$\eta = \frac{\nu}{1 + \alpha} .$$

In lattices that contain no U^{238} or other resonance absorbers, and in which the fuel is fully enriched uranium, there is, in general, no appreciable capture until the neutrons have been slowed down from their initial kinetic energy of some MeV to the "thermal" kinetic energies of the lattice nuclei, the average of which is about 0.025 eV. In this case, the capture-escape probability is therefore simply the fraction of thermal neutrons that become available to the fuel or the "*thermal utilization*" f. For each thermal neutron, $(1-f)$ neutron is absorbed by nuclei other than the fuel, and f is absorbed by the fuel. Thus, in fully enriched lattices, $k_\infty = \eta f$.

The reactor physicist looks up η in a book supplied by the "pure" nuclear physicist and calculates or measures f, by considering the concentrations and the appetites for neutrons (cross sections) of the various nuclei in the proposed lattice.

If the lattice contains appreciable amounts of U^{238}, the story of neutron survival becomes complicated by two effects. A fast virgin neutron can produce a fission in the "non-fissionable" U^{238}.† This results in a dividend of extra neutrons. The calculation of this dividend is complex and depends on the proximity of the nucleus that undergoes fission to the U^{238} target and to the moderator. To avoid writing a complicated formula, this "fast fission effect" is usually accounted for by a factor ϵ introduced into the expression for k_∞.‡

Even more involved is the other complication introduced by U^{238}. This nucleus captures appreciable numbers of neutrons having a kinetic energy intermediate between that possessed by a neutron in the fleeting moment of its virginity and that shared by the neutron with the surrounding matter during its "thermal" life. If the neutrons have a reasonable chance of interacting with U^{238} before they are fully slowed down through collisions with the moderator, there is a substantial chance of their capture in the "resonances" of U^{238}. This chance is designated by $(1-p)$; and p is called "*resonance escape probability*".

A total description of the neutron life cycle in an infinite lattice containing U^{238} can now be given as follows: A fission produces ν neutrons, these are increased by the factor ϵ via fast fissions in U^{238}, reduced before thermalization by the factor p through resonance capture in U^{238}, reduced after thermalization by the factor f through competing thermal capture in non-fissionable nuclei, and reduced by competing thermal capture in the fuel by the factor $1/(1+\alpha)$.

Thus,
$$k_\infty = \nu \epsilon p f \frac{1}{1+\alpha}$$

or,
$$k_\infty = \eta \epsilon p f . \qquad (2)$$

This is the famous "four-factor" formula for k_∞. The thermal capture in the non-fissionable U^{238} is sometimes included in η rather than in f. Thus, the η used for natural uranium metal is usually not that of U^{235} but a synthetic quantity that involves, besides the ratio of capture to fission in U^{235}, the ratio of capture in 99.3% U^{238} to fission in 0.7% U^{235}.

By inserting (2) into (1) we obtain

$$k = \frac{\eta \epsilon p f}{1 + M^2 B_g^{\,2}} . \qquad (3)$$

To answer our crucial question, the reactor physicist must evaluate, calculate or measure the geometric buckling, the migration area M^2, the neutron reproduction factor η, the fast fission factor ϵ, the thermal utilization f, and the resonance escape probability p. Actually, things are even more involved than described here. For example, the distribution in energy of the neutrons, the neutron spectrum, affects the nuclear parameters that enter into η.

It appears that each new lattice requires a considerable number of calculations and/or measurements. However, if there existed a single quantity that combined M^2, η, ϵ, p and f, in such a way that the crucial question could be answered quickly and directly, only a single measurement might be needed. Fortunately, there is such a quantity. It is the "*material buckling*".

4. Material Buckling

Let us consider the case in which a reactor is precisely critical. In that case, the pile dimensions $(B_g^{\,2})$ are such that the particular combination of $B_g^{\,2}$ and of the lattice parameters η, ϵ, p, f, and M^2, that is shown in equation (3) makes $k = 1$. In that case, and only then

$$1 + M^2 B_g^{\,2} = \eta \epsilon p f.$$

We may generalize this relation by introducing a new concept, the "material buckling" $B_m^{\,2}$ such that the equation

$$1 + M^2 B_m^{\,2} = \eta \epsilon p f$$

holds, no matter what k is. Thus, $B_m^{\,2}$ is merely shorthand for

†To cause fission in U^{238} a neutron must have kinetic energies above 1 MeV. The term "fissionable" is commonly applied only to nuclides that can be made to undergo fission with thermal neutrons.

‡$(\epsilon - 1)$ is the dividend rate accruing from fast fission.

$$B_m{}^2 = \frac{\eta\epsilon pf - 1}{M^2} \qquad (4)$$

$B_m{}^2$ happens to be equal to $B_g{}^2$ when $k = 1$, but, in general, it need not be.

By introducing this shorthand into equation (3) we may now write the generally valid equation

$$k = \frac{1 + M^2 B_m{}^2}{1 + M^2 B_g{}^2} \quad .$$

Our crucial question whether k is greater or equal to 1 can now be replaced by the equivalent question whether $B_m{}^2$ is greater or equal to $B_g{}^2$. If $B_m{}^2$ is greater than $B_g{}^2$, k is greater than 1; if $B_m{}^2 = B_g{}^2$, $k = 1$; if $B_m{}^2$ is less than $B_g{}^2$, k is less than 1.

This problem may be compared with the problem of fitting a lady customer. If the dress (geometric buckling) fits the lady (material buckling) the situation is critical for the husband's pocketbook. If the lady is too small for the dress, the matter is inconsequential. If she is too big, the experiment may result in an accident.

5. Material buckling is directly measurable heading:

5. The Measurement of Material Buckling

We have seen that $B_m{}^2$, the material buckling, is a certain combination of properties of the infinite lattice, while $B_g{}^2$, the geometric buckling, is a property of the surface enclosing the finite lattice. The concept of the material buckling is attractive because it reduces the criticality question to a simple comparison of $B_m{}^2$ and $B_g{}^2$, quantities that are different in concept but are similar in their physical dimensions (length^{-2}) and can be expressed in the same units.

Material buckling is directly measurable, thereby obviating separate determinations of M^2 and of the factors entering k_∞. Of course, a more detailed knowledge of reactor performance requires a variety of information beyond the question of initial criticality. Problems of reactor stability, reactivity lifetime, and productivity depend on other combinations of the lattice parameters discussed above, and on still other parameters. The reactor physicist measures, or calculates, many things besides buckling.

Buckling measurements are made according to two principal methods, in critical or in exponential facilities.

In the "critical" experiments, k is very precisely made equal to unity, by equating the unknown material buckling and the geometric buckling. In critical experiments with liquid moderator it is possible to achieve this equality by varying the geometric buckling through adjustment of the liquid level. In situations where the geometric buckling is fixed, the equality is achieved by modifications of the material buckling of the unknown lattice. This can be done by changing, by known amounts, some or several of the reactor parameters that enter into $B_m{}^2$ (see equation 4). For example, f (and M^2) may be adjusted by the addition to the reactor, or removal from the reactor, of poisons that compete for neutrons with the fuel, e.g., by means of control rods. In more sophisticated experiments, a sample of the unknown lattice is inserted into a host lattice of known material buckling, and the reactor is adjusted to criticality. The material buckling of the unknown can then be found by solving a set of equations.

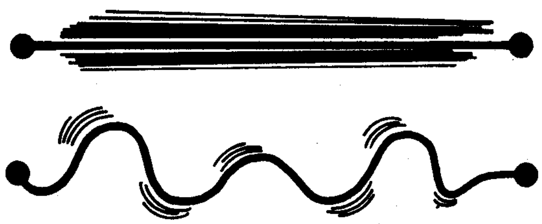
In the "exponential" experiments, it is possible to determine material buckling in a subcritical sample of the lattice and therefore without recourse to geometric buckling. Since this sample cannot maintain a chain reaction, neutrons must be fed into the lattice. The most common arrangement is a cylindrical sample supplied by neutron sources arranged across the bottom surface. (However, Fermi's original exponentials were parallelepipeds.) In such an arrangement, one measures the distribution of neutrons throughout the lattice. The rates at which the neutron population decreases as one proceeds away from the source, and as one approaches the surfaces of the lattice, determine a unique value of $B_m{}^2$. In the conventional vertical tank, it is usually sufficient to measure the radial neutron distribution at one or two levels and the vertical neutron distribution at one or two radii. A plot of neutron density along a vertical axis may be fitted by exponential functions, hence the term "exponential" facility.

Once the material buckling is known, the critical size of the lattice can be derived for any desired shape. This is particularly helpful in problems of nuclear safety in connection with the storage and handling of many fuel pieces. For example, cylindrical fuel slugs could be arranged in many different ways: like bamboo sticks, like soldiers on the drilling ground, or in pyramids like the cans in some grocery stores. From a single material-buckling measurement in an exponential, the critical sizes of any of these configurations can be evaluated, whereas the corresponding critical measurements would require assemblies in each of these configurations.

6. Semantics of Buckling

My dictionary defines the noun "buckle" as "a distortion, as a bulge, bend, kink, or twist in a beam. . .", all of which sounds akin to the problem of fitting the lady customer.

Actually, buckling is related to an eigenvalue problem. The second-order differential equation that maps out the shape of a string or of a membrane in an operating musical instrument involves the local inertia (mass density) and tension of the string or membrane. Solutions are subject to the condition that the ends of the string or the edge of the membrane are in a fixed



position. A similar differential equation describes the neutron density distribution in an operating reactor and involves local lattice properties in a combination called "buckling", as exemplified by our equation (4) defining $B_m{}^2$. The solutions are subject to the condition that the flux must approach zero along the boundary of the reactor. This is true only if the average buckling assumes certain values (the eigenvalues) which involve the information entering into our $B_g{}^2$.

If the buckling is zero somewhere in an operating reactor, the flux distribution is "flat" in this region, that is, the flux changes with constant slope, for example with zero slope. Where the buckling is positive, the flux shape displays curvature and may have a peak, where the buckling is negative there may be a flux depression. So here is some analogy with the "distortions or bulges" in the vibrating string or membrane.

To some of our colleagues, the word buckling is repulsive. They prefer to talk about "the Laplacian", which is not a fortunate choice as it confuses the concepts of differential operator and of eigenvalue.

It was Professor J. A. Wheeler who introduced the term "buckling" into reactor physics. In geometrodynamics, a branch of physics, in which Wheeler is a leading pioneer, the *material* world is reduced to *geometry*.